Einsendeaufgaben – Lektion 6

Modul 61111: Mathematische Grundlagen

Aufgabe 6.3

a)

$$f(x) = \sqrt[3]{x}$$

$$f'(x) = \frac{1}{3} \cdot \left(\frac{1}{\sqrt[3]{x}}\right)^2$$

$$f''(x) = \frac{1}{3} \cdot \left(-\frac{2}{3}\right) \cdot \left(\frac{1}{\sqrt[3]{x}}\right)^5 = -\frac{2}{9} \cdot \left(\frac{1}{\sqrt[3]{x}}\right)^5$$

$$f'''(x) = -\frac{2}{9} \cdot \left(-\frac{5}{3}\right) \cdot \left(\frac{1}{\sqrt[3]{x}}\right)^8 = \frac{10}{27} \cdot \left(\frac{1}{\sqrt[3]{x}}\right)^8$$

Entwicklungspunkt: 27

$$f(27) = 3$$
$$f'(27) = \frac{1}{3^3}$$
$$f''(27) = -\frac{2}{3^7}$$
$$f'''(27) = \frac{10}{3^{11}}$$

$$\begin{aligned} P_{2,27}(30) &= 3 + \frac{1}{3^3} \cdot (30-27) - \frac{1}{3^7} \cdot (30-27)^2 \\ &= 3 + \frac{1}{3^2} - \frac{1}{3^5} \end{aligned}$$

$$|R_{2,27}(30)| = \frac{f'''(\xi)}{3!} \cdot (30-27)^3 < \frac{\frac{10}{3^{11}}}{3!} \cdot 3^3 = \frac{5}{3^9}$$

b)

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x}$$
$$f''(x) = -\frac{1}{x^2}$$
$$f'''(x) = \frac{2}{x^3}$$

Entwicklungspunkt: 1

$$f(1) = 0$$
$$f'(1) = 1$$
$$f''(1) = -1$$
$$f'''(1) = 2$$

$$\begin{aligned} P_{2,1}\left(\frac{5}{6}\right) &= -\frac{1}{6} - \frac{1}{2} \cdot \left(-\frac{1}{6}\right)^2 \\ &= -\frac{1}{6}\left(1 + \frac{1}{12}\right) = -\frac{1}{6} \cdot \frac{13}{12} = -\frac{13}{72} \end{aligned}$$

$$\left|R_{2,1}\left(\frac{5}{6}\right)\right| = \left|\frac{f'''(\xi)}{3!} \cdot \left(-\frac{1}{6}\right)^3\right| < \left|\frac{2}{3!} \cdot \left(-\frac{1}{6}\right)^3\right| = \left|\frac{1}{3} \cdot \left(-\frac{1}{216}\right)\right| = \frac{1}{648}$$